

TRANSLATION

On the Theory Of the Transfinite

Correspondence of Georg Cantor and J.B. Cardinal Franzelin

(1885-1886)

GEORG CANTOR (1845-1918), MATHEMATICIAN AND PHILOSOPHER, carried on an extensive correspondence, on a wide variety of topics, with his colleagues and many others in various countries. After his death, twenty letterbooks were found, into which he had copied his numerous letters. Seventeen of these letterbooks were burned as fuel shortly after the war, and only three were rescued from the flames.

The following correspondence with J. Bapt. Cardinal Franzelin (1816-1886) is contained in these letterbooks. Two of Cantor's letters and a part of Franzelin's reply were published by Cantor himself and incorporated into his work "Mitteilungen zur Lehre vom Transfiniten" ("Communications on the Theory of the Transfinite").

In 1869, Pope Pius IX called a Vatican Council. Without debating here the issues of this council, it is important to note that the convening of the council created an uproar in Europe and especially within international Freemasonry, which convened an opposing council in Naples, in which the "Mazzini networks," including Giuseppe Garibaldi and Victor Hugo, participated. At the Vatican Council the standpoint of the encyclical "De Fide Catholica"—that man can know God through reason—was affirmed. Cardinal Franzelin played an important role in this part of the council, and later in the formulation of the social policies of Pope Leo XIII.

With his first letter to Cardinal Franzelin, Cantor included a brief essay, which has been included in this translation. It is almost identical to an 1885 letter he had sent to his Swedish colleague in Stockholm, Mr. Eneström, and was published by Cantor himself in 1890 in the "Journal of Philosophy and Philosophical Critique." We have also translated several brief, related items from Cantor's correspondence with others.

This is the first time that the complete known correspondence between Georg Cantor and Cardinal Franzelin has been translated into English and published in one location.

The translation of these letters was prepared from the German texts published in *Georg Cantor: Briefe*, edited by Herbert Meschkowski and Winfried Nilson (Berlin: Springer-Verlag, 1991) (GCB) and *Georg Cantor: Gesammelte Abhandlungen mathematischen und philosophischen Inhalts*, edited by Ernst Zermelo (Berlin: Springer-Verlag, 1990) (GCGA). They are published by permission of Springer-Verlag.

Letter from Georg Cantor
to Cardinal Franzelin*
Halle, Germany
December 17, 1885

Permit me, Monsignore, to present to you herewith a small essay (in proof sheet), of which I will take the liberty to send you several copies by book-post, as soon as the printing shall be completed.

I would be pleased, if the attempt contained therein, to properly differentiate the *three main questions* respecting the Actual-Infinite, would also be submitted to examination from the standpoint of the Christian-Catholic philosophers.

The fact that Your Eminence in your great work on dogma, namely in the book "De Deo uno secundum naturam" in thesis XLI does not necessarily reject the standpoint taken by me, which *affirms* the A.I. *in all three* main respects, motivated me already one year ago to take the liberty to inform Your Eminence of my relevant works.

Please accept, Your Eminence, the expression of my greatest esteem, with which I have the honor to sign myself as

very respectfully,
Your Eminence's most loyal
G.C.

*GCB, letter #99, p. 252. Italics indicate author's emphasis only.

On the Various Standpoints
With Regard to the Actual Infinite*

(From a letter by the author to Mr. G. Eneström
in Stockholm on November 4, 1885.)

. . . Your letter of Oct. 31 of this year which I received today contains the following question: [in French—ed.] "Have you seen and studied the essay by the Abbot Moigno entitled: 'Impossibilité du nombre actuellement infini; la science dans ses rapports avec la foi.' (Paris, Gauthier-Villars, 1884)?"¹ Indeed I did obtain this short paper some weeks ago. What Moigno says here about the alleged impossibility of the actual infinite numbers, and the use which he makes of this false argument for the foundation of certain religious doctrines, was already essentially known to me from Cauchy's: "Sept Leçons de physique générale" (Paris, Gauthier-Villars, 1868).² Cauchy seems to have been led to this speculation, most peculiar for a mathematician, by the study of P. Gerdil. The latter (Hyacinth Sigmund, 1718-1802) was a notable, very respected personality and a distinguished philosopher, who worked for a while as a professor in Turin,

afterwards was educator of the subsequent King Karl Emanuel IV of Piedmont, was then called to Rome in 1776 by Pope Pius VI, was employed in various businesses of the Holy See, and finally was appointed Bishop of Ostia as well as Cardinal. Perhaps he will be known to you as the author of some works on geometry and historical matters. Cauchy on page 26 refers to a treatise of Gerdil's, which bears the title: "Essai d'une démonstration mathématique contre l'existence éternelle de la matière et du mouvement, déduite de l'impossibilité démontrée d'une suite actuellement infinie de termes, soit permanents, soit successifs." (Opere edite ed inedite del cardinale Giacinto Sigismondo Gerdil, t. IV, p. 261, Rome, 1806).³ The same subject is also presented by him in "Mémoire de l'infini absolu considéré dans la grandeur" (ibid., t. V. p. 1, Rome, 1807).⁴

I am by no means in fundamental opposition to these authors, inasmuch as they strive for a harmony between faith and knowledge, but I consider the means, of which they avail themselves here to that end, to be entirely wrong.

If the religious dogmas would require for their support such an *absolutely false* principle, as that of the impossibility of actual infinite numbers (which in its well-known formulation "numerus infinitus repugnat"⁵ is as old as the hills; recently it can be found for example in Tongiorgi: "Instit. philos., t. II, 1. 3, a. 4, pr. 10" in the form of: "Multitudo actu infinita repugnat"⁶; it can also be found among others in Chr. Sigwart "Logik, Vol. II. p. 47, Tübingen, 1878," and in K. Fischer "System der Logik und Metaphysik oder Wissenschaftslehre, p. 275, Heidelberg, 1865"),⁷ then they were in a very bad condition, and it seems to me most noteworthy that the holy Thomas of Aquinas in I p. q. 2, a. 3 of his "Summa theologiae," where he proves the existence of God with five arguments, makes *no* use of this faulty principle, although in other respects he is no opponent of the same; in any case it seemed to him at least too uncertain for this purpose. (Compare Constantin Gutberlet: "Das unendliche metaphysisch und mathematisch betrachtet," Mainz, 1878, p. 9.)⁸ As much as I value Cauchy as a mathematician and a physicist, as sympathetic as I find his piety and as much as I am also particularly pleased with that "Sept Leçons de physique générale,"⁹ apart from the error in question, nevertheless I must decidedly protest against his authority, there where he has failed.

It is now exactly two years ago, that Mr. Rudolf Lipschitz in Bonn called my attention to a certain passage in the correspondence between Gauss and Schumacher, where the former declares himself against *any* bringing into play of the Actual-Infinite in mathematics (letter of July 12, 1831); I have answered in detail, and have *in this*

point dismissed the authority of Gauss, of which I think so highly in all other respects, as I reject today the testimony of Cauchy and, in my short paper "Grundlagen einer allgemeinen Mannigfaltigkeitslehre," Leipzig, 1883,¹⁰ among others also the authority of Leibniz, who in this question has committed a peculiar inconsistency.

If you would look more closely at the aforementioned short paper (not the translation in the "Acta mathematica," t. II, where only one part therefrom is printed), then you would find that in paragraphs 4-8 I have fundamentally answered all objections, which could be made against the introduction of actual infinite numbers. Although at that time the writings mentioned of Gerdil, Cauchy, and Moigno concerning our subject were not yet known to me, nevertheless the respective sophisms of these authors are refuted just as well, as the petitiones principii of the philosophers so abundantly cited by me there.

All so-called proofs against the possibility of actual infinite numbers, as can be distinctly demonstrated in every case and can also be concluded from general principles, are in the main point faulty thereby, and therein lies their πρῶτον ψεῦδος,¹¹ that they from the outset demand or rather impose upon the numbers in question all properties of the finite numbers, whereas however the infinite numbers on the other side, if they are to be conceivable at all in any form, must, owing to their contrast to the finite numbers, constitute an entirely new species of number, whose character is by all means dependent on the nature of things and is the subject of inquiry, but not of our caprice or our prejudices.

Pascal, as I have seen only recently, has well recognized the questionable if not paradoxical nature of such deductions, as we encounter them with the mentioned authors, and he therefore also declares himself, just as his friend Antoine Arnauld, *in favor of* the actual-infinite numbers, except that he for a different, refutable reason, which I will not take up in further detail here, underestimates the human mind with regard to its power of comprehension of the Actual-Infinite. (Compare

Pascal, "Oeuvres complètes," t. I p. 302-303, Paris, Hachette & Co., 1877; and also: "Logique de Port-Royal," ed. by C. Jourdin, 4^e partie, chap. 1, Paris, Hachette & Co., 1877).¹²

If one chooses to distinctly classify the various views, which have asserted themselves in the course of history with regard to our subject, the *Actual-Infinite* (henceforward for the sake of brevity denoted by A.-I.), then several viewpoints present themselves for that purpose, of which I wish to emphasize only one today.

One can namely call into question the A.-I. in three main respects: firstly, *inasmuch as it is called in Deo extramundo aeterno omnipotenti sive natura naturante*,¹³

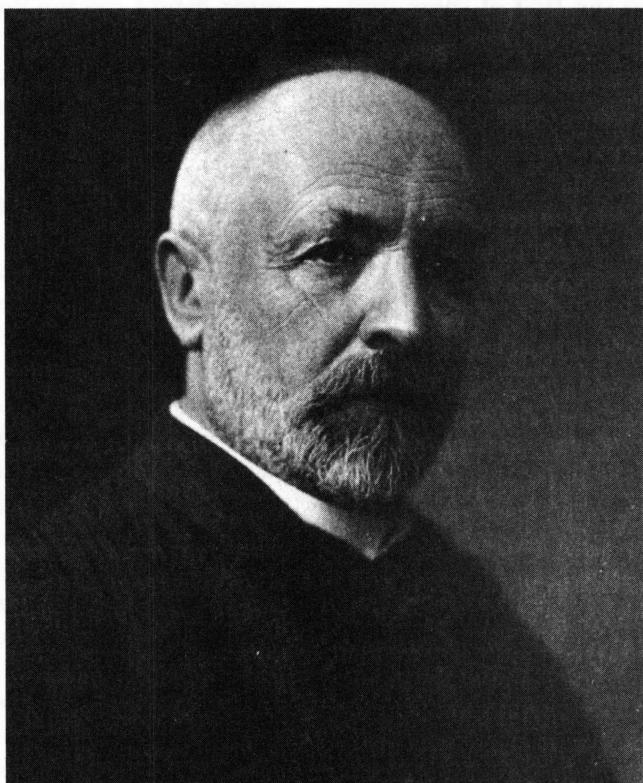
where it is called the *Absolute*, secondly, *inasmuch as it occurs in concreto seu in natura naturata*,¹⁴ where I name it *Transfinatum* and thirdly the A.-I. can be called into question *in abstracto*, that is *inasmuch as it may be comprehended by human cognition [Erkenntnis]* in the form of *actual-infinite*, or as I have named them, *transfinite numbers*, or in the even more general form of the *transfinite ordinal types* ($\alpha_0 \theta \mu_0 \nu_0 \tau_0$ or $\epsilon_0 \delta_0 \gamma_0 \eta_0$).¹⁵

Disregarding the first of these three problems for the moment, and confining ourselves to both of the latter, four different standpoints automatically result, which indeed also find

themselves represented in the past and the present.

One can reject, firstly, the A.-I. not only *in concreto*, but also *in abstracto*, as this is done for example by Gerdil, Cauchy, Moigno in the mentioned texts, by Mr. Ch. Renouvier (compare his "Esquisse d'une classification systématique des doctrines philosophiques," t. I, p. 100, Paris, au Bureau de la Critique philosophique, 1885)¹⁶ and by all so-called *positivists* and their kin.

Secondly, one can affirm the A.-I. *in concreto*, but then reject it *in abstracto*; this standpoint is found, as I emphasized in my "Grundlagen, p. 16,"¹⁷ in Descartes, Spinoza, Leibniz, Locke, and many others. If I have to name here one of the more recent authors, then I mention Hermann



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Georg Cantor

Lotze, who defends the A.-I. in *concreto* in an essay entitled “L’Infini actuel est-il contradictoire? Réponse à Monsieur Renouvier” in the “Revue philos. de Ribot,” t. IX, 1880¹⁸; Renouvier’s reply is found in the same volume of that journal.

Thirdly, the A.-I. can be affirmed *in abstracto*, but then denied *in concreto*; this is the standpoint of one faction of the *neoscholastics*, while another, and perhaps the larger faction of these, a school powerfully spurred by the encyclical of Leo XIII of August 4, 1879: “De philosophia Christiana ad mentem Sancti Thomae Aquinatis Doctoris Angelici in scholis catholicis instauranda”¹⁹ still seeks to defend the first of these four standpoints.

Finally, fourthly, the A.-I. can be *affirmed* not only *in concreto* but also *in abstracto*; on this basis, which I consider the *only right one*, only a few stand; perhaps I am temporally the first, who represents this standpoint with complete determination and in all its consequences, however this I know for certain, that I shall not be the last one who defends it!

Also taking into account the position of the philosophers on the problem of the A.-I. *in Deo*, one obtains a classification of the schools into *eight standpoints*, all of which, strange to say, appear to be represented. One difficulty of the arrangement into these *eight* classes could only result from those authors, who have not taken a definite position with regard to one or more of the *three* questions concerning the A.-I.

The reason that the so-called *potential* or *syncategorematic*²⁰ *Infinite* (*Indefinitum*) gives rise to no such arrangement, is, that it has significance exclusively as a *correlative concept* [*Beziehungsbegrieff*], as an *auxiliary mental image* [*Hilfsvorstellung*] for our thinking, but signifies no *idea* in itself; in that role it has certainly proven, through the differential and integral calculus discovered by Leibniz and Newton, its great value as a means of cognition [*Erkenntnismittel*] and an instrument of our mind; it can not claim for itself a more extensive significance.

Perhaps you were led to pose your question by a remark in my essay “Über verschiedene Theoreme aus der Theorie der Punktmengen,”²¹ in “Acta mathematica,” t. VII, p. 123, where I named among others Cauchy as the authority for my view with regard to the *constitution of matter*; by doing so, I have had in mind especially that component of my hypothesis in which I affirm the *strict spatial point-like quality* [*Punktualität*] or *dimensionlessness* [*Ausdehnungslosigkeit*] of the *last elements*, as they were also taught, following the precedent of Leibniz, by Pater Bosković, in his paper “Theoria philosophiae naturalis redacta ad unicam legem virium in natura existentium, Venetiis, 1763”²²; and certainly this view of Cauchy

is found in his “Sept Leçons,” and is skillfully defended prior to him by André Marie Ampère (*Cours du collège de France* 1835-1836), after him by de Saint-Venant (Compare his “Mémoire sur la question de savoir s’il existe des masses continues, et sur la nature probable des dernières particules des corps.” “Bulletin de la Société philomatique de Paris,” 20 Janvier 1844²³; as well as his larger work in the “Annales de la Société scientifique de Bruxelles,” 2^e année), among us in Germany principally by H. Lotze (compare his “Mikrokosmos,” Vol. I) and by G. Th. Fechner (compare his “Über die physikalische und philosophische Atomlehre,” Leipzig, 1864).²⁴ On the other hand I can not deny that Cauchy at least in that short paper (and indeed also the remaining above-mentioned authors, with the exception of Leibniz) polemicize against the second component of my hypothesis, the *actual-infinite number of the last elements*; with what justification, I have indicated above. That Cauchy nevertheless *on other occasions* did not remain faithful to this opinion respecting the A.-I., as it really could not be otherwise, I will demonstrate some time later. . . .

Despite the essential difference between the concepts of the *potential* and *Actual Infinite*, in that the former signifies a *changeable* finite magnitude, *growing* beyond all finite boundaries, the latter a *fixed in itself, constant* Quantum, situated however beyond all finite magnitudes, it happens to be the case, unfortunately only too often, that the one is confused with the other. Thus for example, the not seldom occurring conception of the *differentials*, as if they were *specific* infinitely small magnitudes (while they are, after all, only *changeable* auxiliary magnitudes, assumed to be as small as you please, which completely disappear from the end results of the calculations and therefore are characterized already by Leibniz as mere *fictions*, for example in Erdmann’s edition, p. 436) is based on a confusion of these concepts. If, however, out of a justified aversion against such an *illegitimate A.-I.*, a certain *Horror Infiniti*, which found its classic expression and support in the mentioned letter of Gauss, has been formed in broad layers of science, under the influence of the modern Epicurean-materialistic tendency of our time, so the therewith connected uncritical rejection of the *legitimate A.-I.* seems to me to be no trifling offense against the nature of things, which one has to take as they are, and this behavior can be understood as a kind of *shortsightedness*, which deprives one of the possibility to see the A.-I., although it in its Supreme, Absolute Bearer has created us and preserves us, and in its secondary, transfinite forms surrounds us everywhere [allüberall] and even dwells in our mind.

Another frequent *confusion* occurs with the two forms of the *Actual Infinite*, in that namely the *Transfinite* is mixed

up with the *Absolute*, while however these concepts are strictly separated, insofar as the former is to be conceived as an *indeed Infinite*, but nevertheless a *yet increasable*, the latter however essentially as *unincreasable* and therefore mathematically *indeterminable*; we encounter this mistake, for example, in *pantheism*, and it constitutes the *Achilles' heel* of *Spinoza's Ethics*, about which, of course, F.H. Jacobi has maintained that it could not be refuted with rational arguments. One can also observe that since Kant, the false notion has come into vogue among philosophers, as if the *Absolute* were the ideal boundary of the *Finite*, while in truth this boundary can only be thought of as a *Transfinitum* and indeed as the *minimum of all Transfinites* (corresponding to the *smallest suprafinite* [überendlichen] number, denoted by me with ω). Without serious critical prior discussion the concept of infinity is treated by Kant in his "Kritik der reinen Vernunft,"²⁵ in the chapter on "Antinomien der reinen Vernunft,"²⁶ in four questions, so as to furnish proof [Nachweis], that they could be *affirmed* or *denied* with equal rigor. It is likely that hardly ever, even taking into consideration the Pyrrhonic and academic skepticism, with which Kant has so many points in common, has more been done for the discrediting of human reason and its capabilities, than with this section of the "critical transcendental philosophy." I will demonstrate at some other time, that it is only through a *vague, distinctionless* application of the concept of the Infinite (if in these circumstances one can still speak of concepts at all), that that author has succeeded in gaining recognition for his *antinomies*, and even that, only among those, who like him willingly evade a thorough mathematical treatment of such questions.

At this point I would also like to respond to two attacks, which have been attempted against my works.

Herbart, as is well known, conceives the definition of the Infinite such, that only the *potential Infinite* can be included in it, so as to thereupon base a so-called proof, that the A.-I. would be self-contradictory. He could have just as well defined the conic section as a curve, whose points are all equally distant from a center, in order to support the thereupon based argument against Apollonius of Perga: "There are no conic sections other than the *circle*, and what you there call *ellipse*, *hyperbola* and *parabola* are contradictory concepts." Of such wares are the objections, which the gentlemen *Herbartians* have put forward against my "Grundlagen." (Compare "Zeitschrift f. exakte Philos.", by Th. Allihn and A. Flügel, Vol. 12, p. 389.)²⁷

Mr. W. Wundt refers, although in a peculiar way, to my works in two of his papers, in his "Logik, Vol. II," as well as in the treatise "Kants kosmologische Antinomien und das Problem der Unendlichkeit, Philos. Studien, Vol. II,"²⁸

and in them the words *introduced by me* "transfinite = suprafinite" [überendlich] stand out frequently; nevertheless I can not find, that he has understood me correctly.

In the *former* work, for example, the whole sentence at the bottom of page 127 which starts with the words: "Wenn wir eine . ." states the exact *opposite of what is correct*. Also the concepts of the *potential* and *Actual Infinite* (which I have called *non-genuine-Infinite* [Uneigentlich-Unendliches] and *genuine-Infinite* [Eigentlich-Unendliches] in my "Grundlagen") are defined by him entirely incorrectly. The juxtaposition with Hegel must likewise be rejected as incorrect. The *pantheistic* Hegel knows no essential differences in the A.-I., whereas it is indeed exactly *my unique characteristic*, to have sharply emphasized such differences, which I found, and to have rigorously mathematically developed them through discovery of the *fundamental* opposition of "*power*" [Mächtigkeit] and "*ordinal number*" [Ordnungszahl] among sets, which Mr. Wundt seems to have entirely overlooked, although it stands out on almost every page of my works. My inquiries bear just as little resemblance to the "mathematical," with which they are nevertheless placed in the same category by Mr. Wundt. The fluctuation of concepts and the confusion connected therewith, which were introduced into philosophy some *one hundred* years ago, at first from the far east of Germany,²⁹ manifest themselves nowhere more clearly than in the questions concerning the *Infinite*, as we see in the innumerable many publications of our modern philosophical literature, be they *criticalistic* or *positivistic*, *psychologicalistic* or *philologicalistic*. Thus it can not remain unmentioned, that Mr. Wundt wishes to use the word "Infinitum" exclusively to signify the potential Infinite. Now this word of old has been *quite generally* related to the most positive of all concepts, that of God; one must be astonished at the peculiar fancy, according to which the word "Infinitum" should henceforth be used only in the most restricted, syncategorematic sense.

EDITOR'S NOTES

1. "Impossibility of the actual infinite numbers; science in its relationships with faith".
2. "Seven lectures on general physics".
3. "Essay on a mathematical demonstration against the eternal existence of matter and motion deduced from the proven impossibility of an actually infinite series of terms, whether continuous or successive".
4. "Memorandum on the absolute infinite considered in magnitude".
5. "an infinite number is contradictory".
6. "an infinite multitude is in fact contradictory".
7. K. Fischer, "System of Logic and Metaphysics or the Theory of Learning".
8. "The Infinite Considered Metaphysically and Mathematically".
9. See footnote 2.
10. "Foundations of a General Theory of Manifolds".

11. "chief deception".
12. Pascal, "Complete Works".
13. "in God—who is Beyond the World, Eternal, Omnipotent—who gives rise to nature".
14. "or concretely, in created nature".
15. "numbers of the mind" or "seen in the eye of the mind".
16. "Outline of a Systematic Classification of Philosophical Doctrines".
17. See footnote 10.
18. "Is the Actual Infinite contradictory? Response to Mr. Renouvier".
19. "Aeterni Patris (On the Restoration of Christian Philosophy)".
20. *syncategorematic*, connoting another idea to express its full meaning; as, the term "son" is *syncategorematic* of the term "father".
21. "On Various Theorems of the Theory of Point Sets".
22. "Theory of Natural Philosophy Reduced to a Single Law of Powers in the Nature of Existences".
23. "Memorandum on the question of determining if continuous masses exist, and on the probable nature of the last elements of bodies".
24. "On Physical and Philosophical Atomic Theory".
25. "Critique of Pure Reason".
26. "Antinomies of Pure Reason".
27. Th. Allihn and A. Flügel, in the "Journal of Exact Philosophy".
28. "Kant's Cosmological Antinomies and the Problem of Infinity".
29. Kant taught in the city of Königsberg, located in what was at that time the far east of Germany.

*GCGA, "Über die verschiedenen Standpunkte in bezug auf das aktuelle Unendliche," pp. 370-376.

Letter from Cardinal Franzelin to Georg Cantor*

December 25, 1885

I am very much obliged to Mr. G. Cantor for the transmittal of the papers about the "Actual Infinite." What greatly pleases me is that the selfsame appears to take not a hostile, but indeed a favorable position with regard to Christianity and Catholic principles. May God the truly Infinite reveal to him the sole necessary truth for finite salvation. I can little busy myself at present with metaphysical discussions; I confess however, that in my opinion, that which the author calls the "Transfinitum in natura naturata," can not be defended, and in a certain sense, although the author does not appear to intend it, would contain the error of pantheism.

*GCB, p. 253.

Letter from Georg Cantor to Cardinal Franzelin*

Halle
January 22, 1886

To His Eminence Cardinal J. Bapt. Franzelin, S.J. in Rome.

The lines, which Your Eminence had the kindness to direct to me on Dec. 25, 1885, contain some doubts with

regard to the philosophical foundation of my works, sent to you for your examination; there are probably certain words used by me whose meaning I have not explained more precisely, which do not bring out my opinion quite exactly, and I would like to take the liberty to briefly explain myself more precisely.

1. I employ the expressions "natura naturans" and "natura naturata" found in my small essay "On the Various Standpoints With Regard to the Actual Infinite" with the same meaning which the Thomists have given to them, so that the *first* expression signifies God, standing outside of the substances created by Him out of nothing, as the Creator and Preserver of the same; the *latter* expression, on the other hand, signifies the world created through Him. Correspondingly I distinguish an "Infinitum aeternum sive Absolutum," which refers to God and His attributes, and an "Infinitum creatum sive Transfinitum," which will be expressed everywhere there, where in the *natura creata* an Actual Infinite must be confirmed, as for example with respect to, in my strong conviction, the actual infinite number of created individual beings, not only in the universe but also already on our earth and, in all probability, even in every ever-so-small extended part of space, wherein I completely agree with Leibniz. (*Epistola ad Foucher*, t. 2 operum, p. I., p. 243). Although I know that this theory of the "Infinitum creatum" is attacked, certainly not by all, but by most church doctors, and in particular, opinions contrary to it are brought forward even by the great St. Thomas Aquinas in his "Summa theol.", p. 1., q. 7., a. 4., nevertheless, the reasons, which in this question in the course of twenty years of inquiry, have forced themselves upon me from within and, so to speak, taken me captive, I might add against my will, because in opposition to always highly esteemed tradition, are stronger than everything which I have hitherto found said against them, although I have investigated it to a great extent. Likewise, I believe that the words of the Holy Scripture, as, for example, in Sap. c. 11, v. 21 "Omnia in pondere, numero et mensura dispositi" ["You have disposed all things by measure, number, and weight." Wisdom 11:20—ed.], in which a contradiction against the actual infinite numbers was suspected, do not have this meaning; for let us suppose, there were, as I believe to have proven, actual infinite "powers" [Mächtigkeiten], that is cardinal numbers, and actual infinite numbers [Anzahlen], that is ordinal numbers (which two concepts, as I have discovered, are extraordinarily different in actual infinite sets, while their difference in finite sets is hardly noticeable), which just as the finite numbers obey strict laws given by God, so quite undoubtedly these transfinite numbers were also meant to be included in that holy utterance and therefore, in my

opinion, it may not be used as an argument against the actual infinite numbers, if a vicious circle shall be avoided.

That, however, an "Infinitum creatum," as existent, must be assumed, can be proven in several ways. So as not to delay Your Eminence too long, I wish to limit myself in this matter to two brief indications.

One proof proceeds from the concept of God and concludes first of all from the highest Perfection of God's Being the possibility of the creation of a Transfinitum ordinatum, then from His Benevolence and Magnificence the necessity of the actually ensued creation of a Transfinitum.

Another proof shows a posteriori, that the assumption of a Transfinitum in natura naturata renders possible a better, because more perfect explanation of the phenomena, especially the organisms and psychical manifestations, than the opposing hypothesis.

The friendly words of appreciation which Your Eminence has spoken with regard to my position towards Catholicism, I owe but little to my own merit, inasmuch as the circumstances into which I am born have had a voice in my standpoint; my highly esteemed late father was indeed Lutheran, my mother, however, whom I have the good fortune to adore among the living, belongs to the Roman Catholic Church and the same is true of her family, as far as I can trace it back. The views, however, which I myself have developed in the course of the years, have never removed me from the fundamental truths of Christianity, but have rather strengthened me therein; I harmonize only very little with the modern philosophical schools, on the contrary I am doing battle with most of them; no system is further removed from my essential beliefs than pantheism, apart from materialism, with which I have absolutely nothing in common.

I believe however, concerning pantheism, that it could be totally overcome in time, and perhaps only through my conception of the matter. Hereby may I be permitted for affirmation of this view to call to mind one of the most gifted pantheists, the German poet *Joh. Wolfgang Goethe*, who shortly before his end, on his last, his eighty-second birthday, August 28, 1831, wrote the following words:

"Long have I resisted,
Finally I give in:
When the old man turns to dust,
The new one will awaken.
And so long as you have not that,
This: die and become!
You are but a gloomy guest
Upon the dark earth."¹

But what concerns materialism and the tendencies connected therewith, as they appear to me, exactly because they are scientifically most untenable and most easily refuted, belong to those evils, of which the human species in the temporal existence shall never be totally freed.

Accept, Monsignore, the expression of high respect and most superior esteem
from Your Eminence's
most devoted servant
Georg Cantor

EDITOR'S NOTE

1. According to Meschkowski, Cantor errs here in attributing these lines to Goethe.

*GCB, letter #100, pp. 254-256.

Letter from Cardinal Franzelin to Georg Cantor* January 26, 1886

Most honored Sir,

From your learned essay "On the Problem of the A.I." I observe with satisfaction how you distinguish very well the Absolute-Infinite and that which you call the *Actual Infinite* in the created. Because you explicitly declare the latter to be a "*yet increasable*" (naturally in indefinitum, that is, without ever being able to become a not more increasable) and set it against the Absolute as "*essentially unincreasable*," which obviously must be just as valid of the possibility and impossibility of reduction or subtraction; thus the two concepts of the Absolute-Infinite and the Actual-Infinite in the created, or Transfinitum, are essentially different, so that when both are compared, only the one must be characterized as *genuine Infinite* [eigentlich Unendliches], the other as non-genuine [uneigentlich] and equivocal Infinite. Perceived thus, as far as I see until now, no danger for religious truths lies in your concept of the Transfinite. Nevertheless, in one respect you most certainly go astray against the unquestionable truth; this error, however, does not follow from your concept of the Transfinitum, but from the deficient conception of the Absolute. In your esteemed letter to me, you say, to wit, at first correctly (provided that your concept of the Transfinitum is not only religiously inoffensive, but also *true*, whereof I do not judge), one proof proceeds from the concept of God and concludes first of all from the highest Perfection of God's Being the possibility of the creation of a Transfinitum ordinatum. On the assumption that your Transfinitum *Actuale* contains no contradiction in itself,

your conclusion of the *possibility of creation* of a Transfinitum out of the concept of God's Omnipotence is entirely correct. My only regret is you go further and conclude "from His Benevolence and Magnificence *the necessity* of an actually ensued creation of the Transfinitum." Exactly because God in Himself is the absolute infinite Good and the absolute Magnificence, which Good and which Magnificence nothing can augment and nothing diminish, the *necessity* of a creation, whichever that may be, is a contradiction, and the *freedom* of creation a just as necessary Perfection of God, as all His other Perfections, or better, God's infinite Perfection is (according to our necessary distinctions) just as well *Freedom*, as Omnipotence, Wisdom, Justice, etc. According to your conclusion of the *necessity* of a creation of the Transfinitum, you ought to go much further yet. Your Transfinitum Actuale is an increasable; now if God's infinite Benevolence and Magnificence really demands with necessity the creation of the Transfinitum, so, for entirely the same reason of the infiniteness of His Benevolence and Magnificence, the necessity of increase until it would be no longer increasable follows, which contradicts your own concept of the Transfinitum. In other words: he who infers the necessity of a creation from the infiniteness of the Benevolence and Magnificence of God, must maintain, that everything creatable is indeed created from eternity; and that before the eye of God there is nothing possible, that His Omnipotence could call into existence. This unfortunate opinion of yours, of the necessity of creation, will very much hinder you, also in your so praiseworthy fight against the pantheists, and at least weaken the persuasive power of your arguments. I have dwelt on this point so long, because I most sincerely wish that your great acumen would free itself from such a fateful error, which of course many other great minds lapse into, even those who consider themselves orthodox.

What you write to me about your position regarding Catholicism, was on the one hand very gratifying, especially when I consider the surroundings within which you find yourself; but on the other hand I can not conceal from you, how painful it is for me, that you have the misfortune of finding yourself outside your mother's house. For men of your position, reflection upon the most important and for eternity decisive concern of religion is necessary, but much more necessary still, is humble prayer for illumination and strength from above.

I am no longer able to engage in a further correspondence about your philosophical views, with my many occupations, through which I am dependent upon an entirely different field; you may thus excuse me, if I will

not be able to answer your possible replies, which however, inasmuch as they refer to your system, I ask you to discontinue.

With respect, most honored Sir
Yours most faithfully
(signed) J B Card. Franzelin

*GCGA, (partial) pp. 385-386. GCB, (partial) pps. 256-257, 511-512 (facsimile, partial).

Letter from Georg Cantor
to Cardinal Franzelin*
Halle
January 29, 1886

Your Eminence, I wish to express my warmest thanks for the expositions in your kind letter of the 26th of this month, with which I agree with full conviction; for in the brief indication of my letter of the 22nd of the same month, it was not my intention at the point in question, to speak of an objective, metaphysical necessity of the act of creation, to which God the *absolute Free* would have been subjugated; on the contrary, I wanted to point to a certain subjective necessity *for us*, to infer from God's Benevolence and Magnificence an actually *ensued (not a parte Dei ensuing)* creation, not only of a *Finitum ordinatum*, but also of a *Transfinitum ordinatum*.

Accept, Monsignore, my most sincere thanks for all the evidence of your fatherly goodwill and your great kindness.

Yours
most respectful devoted
G. C.

*GCB, letter #101, p. 258.

Excerpt from a letter from Georg Cantor
to Gösta Mittag-Leffler*
Halle
Dec. 23, 1883

. . . My good friends, who like to call themselves metamathematicians, may think of my ideas as they will, they may write to London and Paris and for all I care to Kamchatka about what they think is right, I surely know, that the ideas on which I work with my weak powers will engage for generations the thinking minds, even when I and my good friends, the gentlemen metamathematicians, have long gone the path of all mortals. I am far from attributing my discoveries to personal merit, because I am only an instrument of a higher power,

which will continue to work long after me, in the same way as it manifested itself thousands of years ago in Euclid and Archimedes. . . .

*GCB, letter #59, pp. 159-160.

Letter from Georg Cantor
to Professor C.A. Valson*
Halle
Jan. 31, 1886

Professor C.A. Valson, in Lyon, 25 rue du Plat.

Highly esteemed colleague,

I deliberately put off my reply to your kind letter of Jan. 18, '86, because it was my intention to answer in detail; unfortunately I am still too much overloaded with various work and will therefore no longer wait to express to you my courteous thanks for the worthy as well as interesting present of your work on André-Marie Ampère as well as your letter. The "discours préliminaire" in your book will fascinate me no less than the other part, because I, as you know, treasure the value of all efforts which are directed towards elevating science to a more ideal standpoint, than can be achieved through pure rationalism, which through the brilliant talents of a Lagrange, Laplace, Gauss, etc., was led to develop and flower, and from which influence even Cauchy and many other of today's living geometers, whose tendency of heart, if I may say so, leans in a different direction, have not been able to fully escape. There is much I could say about all of this, but I confine myself to just this, that it is my conviction that the great achievement of Newton, the "Principia mathematica philosophia naturalis," to which all of the recent developments of mathematics and mathematical physics have conformed, is to be seen, because of the gross metaphysical shortcomings and erroneousness of his system, despite the good intention of the originator, as the true cause of the materialism or positivism of our time, which has grown into a kind of monster, strutting in the radiant robe of science, especially in the metropolitan and world-famous academies. Thus we see, that the greatest achievement of genius, despite the subjective religiosity of the author, if it is not united with true philosophical and historical spirit, leads to consequences, and I go so far as to declare, must necessarily lead to consequences whereby it is highly questionable, whether the good in them is not far surpassed by the evil which they simultaneously inflict upon mankind; and to the worst of evils it appears to me belong the errors of modern scepticism, which considers itself "positive" and harks back to Newton, Kant, Comte and others. I also wanted to send along some metaphysical theses for exam-

ination by Abbot Ehè Blano, but I must also postpone that until a later date.

Thank you as well for the excerpts from "Fraité de Mécanique de Poisson" about the "infiniment petit." You give me herewith the desired opportunity to declare that there is no more determined opponent of these conceptions of Poisson, which are full of contradictions, than I, and that I most despise this kind of "Infiniment petit ou grand," which I call in the very beginning of the enclosed note the "L'infini actual illegitima"; it has led only to misunderstanding of the "Infini actual légitime." I rather hold that conception of the merely potentially infinite generally found in mathematics, for which especially the extremely significant works of Cauchy paved the way (although in Leibniz already the same conception of the differential is found), to be the *only correct one*. My works pertain to a totally different and in the main point new *mathematical* ordering of ideas, than can be achieved through the Newtonian principles, which, however, until now has only been recognized by a few. They do not refer directly to something above nature; they rather aim at a more precise, more complete, more refined knowledge of *nature itself*, certainly not without contact with Him, who stands above nature, because it is His voluntary creation. Please accept, Sir, the expression of my distinguished esteem and respect.

Your most devoted
(signed) Georg Cantor

PS. Could you perhaps recommend to me a young man who would be enough of a philosopher and mathematician, and would be kind enough to produce for me small appropriate excerpts from texts, which I can not find in Germany, but which might be easily obtained in the libraries of Lyon or Paris? I would be greatly indebted to you.

*GCB, pp. 512-513 (facsimile).

From "Mitteilungen zur Lehre
vom Transfiniten"*

(From a letter from Georg Cantor to A. Eulenberg,
Feb. 28, 1886)

. . . The *Transfinite* with its abundance of formations and forms, points with necessity to an *Absolute*, to the "truly Infinite," to whose Magnitude nothing can be added or subtracted and which therefore is to be seen quantitatively as *absolute Maximum*. The latter exceeds, so to speak, the human power of comprehension and eludes particularly mathematical determination; whereas the *Transfinite* not only fills the vast field of the possible in God's knowledge,

but also offers a rich, constantly increasing field of ideal inquiry and attains reality and existence, I am convinced, also in the world of the created, up to a certain degree and in different relations, to bring the Magnificence of the Creator, following His absolute free decree, to greater expression than could have occurred through a merely "finite world." This will, however, have to wait a long time for

general recognition, especially among the *theologians*, as valuable as this knowledge would prove to be as a resource for the promotion of their domain (religion). . . .

*GCGA, pp. 405-406.

—translated by Gabriele Chaitkin

An Afterword by Lyndon H. LaRouche, Jr.

July 30, 1994

Georg Cantor: The Next Century

The relatively brief period of Halle-to-Rome correspondence between mathematical genius Georg Cantor and Cardinal Johann Baptiste Franzelin, S.J. remains one of the more significant anomalies in the history of science, and also theology. To appreciate the central feature of that correspondence itself, it is essential to identify some crucially relevant features of Cantor's life: then, and during the decade following the termination of that exchange of letters.

Georg Cantor's 1897 *Contributions To The Founding of The Theory of Transfinite Numbers* (*Beiträge*)¹ is an indispensable work; but, there are problems. Cardinal Johann Baptiste Franzelin's abrupt termination, on Jan. 26, 1886, of his ongoing correspondence with Cantor,² is crucial for understanding fully the darkened mood which distinguishes Cantor's writings of the 1890's from those of the 1880's; and that latter period in Cantor's life is one of the keys to understanding the circumstances in which the correspondence was terminated.

Directly to the crucial issue: Cantor's depression confronts the informed reader immediately at the outset of reading the *Beiträge*. Exactly as it is placed there in the 1962 edition,³ the evidence is:

"*Hypotheses non fingo*" [—Newton].

That reference would not have been allowed by the Cantor of the Franzelin correspondence, the 1883-84 *Grundlagen*,⁴ or even the 1887-1888 "Mitteilungen zur Lehre vom Transfiniten."⁵ The Cantor of 1897 and later, pleading for recognition from Britain, and engaging himself in such pathetic enterprises as the myth of Francis Bacon's authorship of Shakespeare's works,⁶ is no longer the Cantor of the 1880's.

This mid-1890's change in Cantor's mood has been misused by sundry sophists as a pretext for deriding not

only the 1897 *Beiträge* as "pathological science," but also such earlier writings as the *Grundlagen*. There are problematic features in the *Beiträge*, but none to which those critics might rightly object. From the vantage-point of those who have studied the more vigorous Cantor writings of the 1880's, the failing of the *Beiträge* is its propitiatory quality, its excessive reliance upon formalism, just as the dedicatory note to Newton might imply.

Since our purpose here is to situate the Cantor-Franzelin correspondence, we are permitted and obliged to dispense with the subsidiary mathematical formalities of the matter as much as possible. Under those circumstances, the immediately following descriptive observation is supplied.

All of the crucial conceptions met in the *Beiträge* are met in earlier writings of the 1883-1888 interval; the significance of the 1897 book is that it supplies a proof and some further elaboration of those conceptions from a strictly formal standpoint. The Georg Cantor of 1897, a mere fifty-two years of age, has become, in one very important sense of the term, "an old man," his enemies have finally succeeded in quenching his creative spark. He is left to no more than commenting faithfully upon the achievements of a brilliant past state of mind, to which he is fated never fully to return. The operative term there is "reporting faithfully"; the discovery reported in the 1897 book is authentically Cantor's, but, sadly, the exposition is added by a Cantor who could no longer make new such original discoveries.

If one takes all the relevant elements of Cantor's environment into account, Cardinal Franzelin's abrupt termination of the correspondence was at least a contributing cause for Cantor's very-premature old age. The Cardinal clearly did not intend such an effect; the problem was, that the topics of that correspondence are the same

issues which mobilized the rogues of the European science community, especially the mathematicians, in a two-decades-long aversive behavioral modification of Cantor. Those topics, which are the essential content of the correspondence, are the issues prompting Leopold Kronecker and his positivist accomplices to conduct one of the most widespread and disgusting inquisitions in the internal history of science, the virtual lynching of Georg Cantor.

Georg Cantor's Theology

Georg Cantor, born of Jewish ancestry in St. Petersburg, Russia on March 3, 1845, began life with a grand heritage. He was the maternal grandnephew of the Joseph Boehm who was, in turn, the collaborator of Ludwig van Beethoven in the performance of Beethoven's late string quartets, who was the founder of the Vienna school of violin performance, and personally the teacher of the famed violinist Joachim. That musical tradition permeated the family; until his adolescent turn into mathematics, Georg Cantor himself was trained as a violinist in this tradition, and two of his siblings, in addition to other immediate relatives, were notable musicians. The family converted to a Protestant rite, and moved to Germany, where he studied in such locations as Wiesbaden and Darmstadt.

During 1885-1886, this Jewish-born German Protestant, and music-student turned mathematical genius, is exchanging correspondence on some of the most profound issues of theology with an influential Cardinal in the Rome of Pope Leo XIII. To cap those ironies, Cantor was by no means unprepared.

This correspondence was prompted, on Cantor's part, by a question addressed to him, asking whether he had seen a certain writing by French Abbot Francois Napoleon Marie Moigno.⁷ This provoked a Nov. 4, 1885 letter to one G. Eneström in Stockholm,⁸ and the enclosure of a copy of that letter in Cantor's letter of Dec. 17, 1885 to Franzelin.⁹ The Cardinal acknowledged this communication in a letter of Dec. 25, 1885, cautiously rebuking Cantor's criticism of Cauchy and Moigno with the suggestion that Cantor might abstain from the appearance of pantheism.¹⁰ To this, Cantor replied on Jan. 22, 1886. The response from the Cardinal was issued on Jan. 26, 1886, excusing himself from further correspondence with Cantor.¹¹ Cantor sent a "thank you" letter for consideration given on Jan. 29, 1886, but received no acknowledgement.¹²

To assess the Cardinal's manifest reaction to Cantor's attack on the characteristically neo-Aristotelian (e.g., positivist) fallacies of Cauchy and Moigno, one must take into account the reputation already gained in profession-

al circles at that time by Cantor's 1883-1884 *Grundlagen*.¹³ This work had mobilized Cantor's enemies into attack at full tilt, led, as always, by Kronecker. Cantor's reaction to the query respecting Moigno's piece, is visibly a response to the already ongoing political lynch-mob being mobilized against him, in Germany, France, and elsewhere.

With the *Grundlagen*'s appearance, it is evident that he is well-grounded in Plato's work, and is attempting to view the method of Leibniz from that standpoint. He has also shown himself a follower of Cardinal Nicolaus of Cusa in these matters. The appearance of the "Mitteilungen"¹⁴ affirms that continuing commitment. This establishes Cantor's scientific and theological outlook very clearly for anyone with the prerequisites to assess this.

Briefly: Cantor himself insists that his science and theology center around two crucial points of equivalence between his own work on the transfinite and Plato's principle of hypothesis. His opinion on these parallels is broadly correct.¹⁵ Cantor insists that his general notion of the *Transfinite* is equivalent to Plato's *Becoming*, and that his own *Absolute* corresponds to Plato's *Good*. By *Becoming* is signified Plato's generalized notion of what Plato terms *hypothesizing the higher hypothesis*.¹⁶ Obviously, to follow the argument in Cantor's letters (or, elsewhere, for that matter) one must first understand what is signified by Plato's principle of *hypothesis*.

For the purposes of formal criticism, especially formal mathematics or mathematical physics, Plato's principle of hypothesis is best presented in terms of his *Parmenides*: the ontological paradox of the *One* and the *Many*. His solution for that paradox is the formal definition of human creativity, as valid axiomatic revolutions in formal mathematical physics typify creativity, in the sense of Cantor's definition of *type*. In Plato, the term *hypothesis* signifies such a type of discovery, and never anything different. Briefly, work through an illustration of Plato's discovery of the principle of hypothesis.

The secondary student's classroom model of reference for a *Many* is Euclid's geometry: an expandable lattice-work of theorems, each and all mutually consistent with one another in terms of a shared, fixed set of axioms and postulates. That expandable list of theorems constitutes a *Many*. The challenge is to identify a single conception such that, when we think about that single conception, we are implicitly defining each and every theorem which might possibly be part of that theorem-latticework. If one adheres to the formalist methods of a *Parmenides*, a Sophist, an Aristotle, a Galileo, a Newton, a Cauchy, a Kronecker, a Bertrand Russell, or a John Von Neumann, no true solution to this ontological paradox is possible.¹⁷

However, let us discover a proposition which is true in nature, but which cannot be consistently a theorem of

that theorem-latticework; let us designate that latter as theorem-lattice “A.” This theorem requires us to alter some part of the set of axioms and postulates of theorem-lattice “A” to the effect that all of the old theorems must now be scrapped in their earlier form, and recalculated on the basis of a new set of axioms and postulates, theorem-lattice “B.” In another case, nature obliges us to proceed to a third theorem-lattice, “C.” On this basis, Plato hints in writing the *Parmenides*, a solution for discovery of the *One* is attainable.

Instead of focussing upon fixed objects, such as sense-objects, one must focus upon *change* itself as the primary fact of nature, and of mental life. In the given case, it is the change from *A* to *B*, and from *B* to *C*, which is crucial. It is this change which one can conceptualize as an unified object of thought, a *One*. This permits us to conceptualize the changes in the respective underlying sets of axioms and postulates, from *A* to *B*, as a unit, as a *One*.

That *One* is an hypothesis. Any valid axiomatic-revolutionary discovery of that *type* is an instance of hypothesis as Plato defines hypothesis.

Next, continue with the illustration provided. Examine the successive changes, from *A* to *B*, *B* to *C*, and, then, *C* to *D*. This sequence of changes—of hypotheses—is a Many, too. Scrutiny of this Many enables us to conceptualize a higher sort of *One*. As the first level of *One*—e.g., *A* to *B*—defined an hypothesis, the new *One* required is a *method of generating hypotheses: a higher hypothesis*. It is a method of discovery. In natural science historically, there is evidence of various types of relatively valid methods of discovery, but some proving more valid than others. Study of the Many alternative, relatively valid choices of methods of hypothesis (higher hypotheses) yields Plato’s *hypothesizing the higher hypothesis*.

That latter, hypothesizing the higher hypothesis, is Plato’s knowledge of the *Becoming*. The notion of a *One* corresponding to a *Many* is Cantor’s notion of a transfinite; he is occupied with examining the general hierarchy of transfinities as a domain defined in the sense indicated by Plato’s principle of hypothesis.

This principle of hypothesis implies the necessary existence of the *Good*. Since hypothesis is development in physical space-time, a *Many*, what is the *One* which corresponds to hypothesizing the higher hypothesis respecting physical space-time? It must be intelligence; it must be all space, all time, combined with efficient (creative) intelligence as *One*. That is Plato’s *Good*; that us what Cantor signifies by *Absolute*.

On this issue, the London-aligned political party within European science was united in a *maenad*’s hateful frenzy, not only against Cantor’s notion of the mathematical transfinite, but also the related work of Karl Weier-

strass, Riemann, *et al.* earlier. This is a continuation of Venice Abbot Antonio Conti’s war to destroy Leibniz and rehabilitate Galileo; this is a continuation of Paolo Sarpi’s use of the “brainwashed” Galileo to guide Bacon *et al.* in their attacks upon Nicolaus of Cusa, Leonardo da Vinci, and Johannes Kepler. This is the issue of 1885–1886, between Cantor, on the one side, and the followers of LaPlace, Cauchy, and Moigno, on the opposing side.¹⁸ This is the mathematical, ontological, and theological issue which permeates the immediate environment of the Cantor-Franzelin exchange.

To identify the axiomatic formalities of the issue between Cantor and such followers of Galileo and LaPlace as Cauchy and Moigno, it is sufficient to focus upon the review of elementary geometry just supplied here. Look at the change in proceeding from the axiomatic basis of theorem-lattice *A* to that of *B*, or *B* to *C*, or *C* to *D*.¹⁹ From the standpoint of Aristotelian formalism, the movement from one such lattice to the higher successor is a formal-logical *discontinuity*, and also a mathematical discontinuity. This discontinuity, separating the axiomatic basis of one theorem-lattice from the next, is the formal reflection of an act; it is the representation of what we term in physics a true *singularity*. That act is the employment of the creative processes of mind, as described by Plato’s Socratic method, to discover a solution to a “*One/Many*” paradox of the type illustrated by the *Parmenides*.

This discontinuity, which has a mathematical size of virtually zero—but not zero, is a correlative of what Plato signifies by “*change*.” This change, this mathematical discontinuity is the root ontological referent for Cantor’s notion of the transfinite. Since Riemann’s famous Habilitation dissertation of 1854 on hypothesis, such singularities expressed as paradoxes of the formal domain of mathematics are the entry-points for the crucial issues of physics, which can be addressed efficiently only from the standpoint of physics, and not formalist mathematics as such.²⁰

In light of this kind of evidence, it is clear than the “*infinite*” as conceived by Aristotle and other formalists does not exist. The proof is, that every formal theorem-lattice, within whose terms such a popular misapprehension of the term “*infinite*” is projected by formal logic, is itself finite or, “*transfinite*”! Every theorem-lattice is bounded externally by a higher-order theorem-lattice, until the very conception of Plato’s *Becoming* reaches its upper, external boundary, defined by the *Good*, the location of existence of the Mosaic God of the Apostles John, Paul, *et al.*, which latter bounds everything efficiently. Those are the mathematical, physics, and theological implications of the Cantor-Franzelin exchange, the environment within which the discussion is situated.

The fact that discovery of relatively higher-order theorem-lattices enables us to conceptualize as a single mental object the differences between the respective sets of axioms underlying two compared formal theorem-lattices, permits us to replace the commonplace, but pathological notion of an “infinite” with the notion of the boundedness, hence “transfiniteness” of that set of axioms which defines the theorem-lattice, within which latter the corresponding pathological notion of an “infinite” is situated.²¹

Cantor’s general form of solution to conceptualization of the notion of infinite in a non-pathological way, is to express the Many-ness of very large arrays within a specific theorem-lattice by a One. That One is the unified notion of the set of axioms and postulates underlying the consistency among all possible theorems of that specific theorem-lattice type.

This is the problem which Bertrand Russell, for one, attempts to circumvent by mere word-juggling, using the term “hereditary principle.” I.e., since every possible theorem of a consistent lattice is hereditarily consistent with the imputable set of axioms and postulates underlying it, that set of axioms and postulates must be construed as an “hereditary principle”; once the hereditary principle’s distinctions are understood, as distinct from that of other lattices, the notion of any infinity apparently existing within a formal lattice is expressed adequately by direct reference to the “hereditary principle.” The trouble with Russell’s version of this, and those of his followers, is that his views involve a deliberate fraud, a methodological, formalist’s fraud closely related to that of LaPlace, Cauchy, and Moigno earlier.

To understand the Cantor-Franzelin exchange adequately, one must know these background considerations. To understand Cantor himself adequately, one must return to the clean fresh air of Riemann’s 1854 paper on hypothesis.

Once one steps out of the precincts of the street mathematician, into the realm of theology, the issue between Cantor and Moigno is a replay of the continuing issue between Cardinal Nicolaus of Cusa and Aristotelian apologist John Wenck, back during the 1440’s. Not only does Cantor rightly trace his discoveries to the mathematical discoveries of Nicolaus of Cusa. That is the issue of attacks on Cusa by Pietro Pompanazzi and his students, such as Francesco Zorzi, and the later attacks upon Cusa’s method and influence by the atheists Paolo Sarpi (who deployed Galileo) and Cauchy’s mentor LaPlace.²² To pose such issues within a theological deliberation among public figures, one a cardinal, in the 1880’s, is to raise the specter of possible schism between the followers of St. Augustine (the Platonists) and the

followers of Wenck and Pomponazzi (the Aristotelians). To say the least, Cantor posed a very touchy subject in his correspondence.

Georg Cantor fully in his right mind would never adopt Newton’s “*hypotheses non fingo*,” nor send praises of Theosophist’s hero Francis Bacon to Pope Leo XIII.

The Formalities of the Issue

Now, to conclude, identify as simply as possible the form of the issue between the followers of LaPlace and Cantor, the formalities of the Cantor-Franzelin exchange.

Cantor’s correspondence references symptomatically an issue which is as old as the beginning of modern European civilization, the issues of the principles of the founding of modern science by Nicolaus of Cusa’s *De Docta Ignorantia*²³ and related writings.

Once one situates observation of the act of mental-creative discovery within the formalities of classical geometry, as Cusa did in solving the ontological paradox of Archimedes’ theorems on quadrature of the circle, one has immediately two notable results. First, one has rendered the act of creative mental activity itself a subject available to conscious reflection, has rendered the creative processes of the mind *intelligible*. One is obliged to explore the same principle of intelligible creativity shown in such a geometry setting, to see the same quality of intelligible mental phenomenon in other areas of application.

Since the work of Paolo Sarpi’s tame gnostic, Galileo Galilei, the fraudulent tactic which the followers of Galileo’s method have employed to attempt to evade the kinds of singularities to which we have referred above, is to insist, hysterically, as Venice agent Dr. Samuel Clarke did in the Leibniz-Clarke correspondence, upon the ultimate authority of infinite series. They claim, that since infinite series may approximate all possible values within mathematical functions, mathematical discontinuities do not exist. Often, they even worship such an infinity, insisting that the unfathomable outer reaches of “infinity” are the place of residence of what Harvard Professor William James specified as the universal common root of “varieties of religious experience,” or what Sigmund Freud (or, is it “Fraud”) identified as “the oceanic feeling.”²⁴

That copying of the notion of infinite series inhering in the method of Galileo, is that same standpoint expressed by Venice’s Eighteenth-Century control agent, Abbot Antonio Conti, his accomplice Abbot Guido Grandi of Pisa, and his protégé and Grandi student Giacomo Ortes. This is the standpoint of radical

empiricism, such as that of Jeremy Bentham and his followers in Britain, and also the standpoint of the French Restoration form of radical empiricism, the positivism of the followers of LaPlace and Cauchy.²⁵

Cardinal Franzelin's abrupt termination of the correspondence with Cantor did not cause Cantor's capitulation to British Theosophy during the late 1890's; unfortunately, had Franzelin's rejection of continued discussion not have occurred as it did, Cantor's mind might not have cracked under the pressures of such London assets in Germany and France as Kronecker and his accomplices.

Cantor's work remains a great contribution to mankind, and his efforts to clarify this issue with a representative of the Vatican are an honorable part of that. His collapse under two decades of his enemies' aversive attempts at his behavioral modification, is an important tragedy of modern history, especially for science, but also for mankind. Cantor himself believed that his discoveries would not be properly appreciated until some time during the Twentieth Century. Generally speaking, his insight on that point was prophetic, although we must thank those, including Kurt Gödel, who kept his work alive for us today. To go forward with his contributions, it is sufficient to begin with a slight detour, to situate Cantor's discoveries within the developments flowing through Riemann's 1854 habilitation dissertation on hypothesis.

NOTES

1. Georg Cantor, *Beiträge zur Begründung der transfiniten Mengenlehre* (1897), in *Georg Cantors Gesammelte Abhandlungen*, ed. by Ernst Zermelo (Hildesheim: Georg Olms Verlag, 1962), pp. 282-351. The readily available English translation is that of Cambridge University-trained Philip E.B. Jourdain: Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers* (1915) (New York: Dover Publications, 1955). For reason of that precedent, the Jourdain English translation of the title has been employed here. The reader is cautioned that Jourdain's notes for the 1915 edition are rendered obsolete by Kurt Gödel's "On formally undecidable propositions of *Principia Mathematica* and related systems I" ("Über formal Unentscheidbare Sätze der *Principia Mathematica* und verwandter Systeme I"), in *Kurt Gödel: Collected Works, Vol. I*, ed. by Solomon Pfeferman et al. (New York: Oxford University Press, 1990), pp. 144-199 (including appended note by editors).
2. On Cardinal Franzelin's termination of the correspondence, see *Georg Cantor Briefe*, ed. by Herbert Meschkowski and Winfried Nilson (Berlin: Springer-Verlag, 1991), pp. 256-257. On the subject of this correspondence and also Cantor's depression of the 1890's, see the same source, pp. 11-16, 252-258, 282-285.
3. *Op. cit.*, p. 282.
4. *Grundlagen: über unendliche lineare Punktmannigfaltigkeiten*, in *Gesammelte Abhandlungen*, *op. cit.*, pp. 139-246.
5. *Op. cit.*, pp. 378-451 (including appended notes from Dedekind correspondence).
6. See Meschkowski and Nilson, *op. cit., passim*. The Anglophilie phase of Cantor's depression erupts visibly during the approximately two-year span of time from the 1895 break in his already deeply strained intellectual relationship with Professor Felix Klein, through such 1897 events as the publication of the *Beiträge* and the death of Cantor's former mentor, Karl Weierstrass. During that interval, Cantor has developed a close acquaintance with Rudolf Steiner, a member of the British Theosophist movement, a founder of the Vienna-based Theosophist periodical, *Luzifer*, and later founder of the German (Waldorf) spin-off of the Theosophists, the Anthroposophic movement. (The legend is that Steiner concluded that the radicalism of Bertrand Russell's crony, the Theosophical leader and satanist Aleister Crowley, was a bit strong for customary German Kantians, and produced the altered dogma of the anthroposophs with this thought in mind.)
7. *Impossibilité du nombre actuellement infini; la science dans ses rapports avec la foi* (Paris: Gauthier-Villars, 1884). See Cantor, "Über die verschiedenen Standpunkte in Bezug auf das aktuelle Unendliche," in *Gesammelte Abhandlungen*, *op. cit.*, pp. 370-377.
8. *Ibid.*
9. Meschkowski, *op. cit.*, pp. 252-253.
10. *Ibid.*
11. *Op. cit.*, pp. 254-257.
12. *Op. cit.*, p. 258.
13. *Op. cit.*
14. *Op. cit.*
15. Cantor's repeated insistence on this during his writings of the 1880's is indispensable for avoiding the commonplace blunders of the proverbial "usual generally recognized authorities" in their reading of both the *Beiträge* and these earlier writings.
16. There is a presentation of this in numerous of this author's writings, including Section 2 of the current "How Bertrand Russell Became An Evil Man," *Fidelio*, this issue, pp. 33-73.
17. Kurt Gödel, *op. cit.*
18. LaRouche, "Evil Man," *op. cit.*; Section 2, *passim*.
19. *Ibid.*
20. *Ibid.*
21. *Ibid.*
22. *Ibid.*
23. Nicolaus of Cusa, *De Docta Ignorantia (On Learned Ignorance)* (1440) [trans. by Jasper Hopkins as *Nicholas of Cusa on Learned Ignorance* (Minneapolis: Arthur M. Banning Press, 1985)].
24. LaRouche "Evil Man," *op. cit.*; Section 2, *passim*.
25. *Ibid.*